Int. J. Heat Mass Transfer. Vol. 15, pp. 1225–1242. Pergamon Press 1972. Printed in Great Britain

COUPLED HEAT AND MASS TRANSFER IN REGENERATORS—PREDICTION USING AN ANALOGY WITH HEAT TRANSFER

I. L. MACLAINE-CROSS

Department of Mechanical Engineering, Monash University, Clayton, Victoria, Australia

and

P. J. BANKS

Division of Mechanical Engineering, Commonwealth Scientific and Industrial Research Organization, Highett, Victoria, Australia

(*Received* 1 *December* 1970 *and in revised form* 16 *August* 1971)

Abstract—In regenerative heat and mass exchangers, or regenerators, heat and one or a number of adsorbates are transferred cyclically from one fluid stream to a porous matrix and then from the porous matrix to the other fluid streams. In the simple model used here transfer is described by a single film transfer coefficient, and diffusion in the fluid flow direction and sorption hysteresis are neglected. The equations describing the behaviour of this model are non-linear coupled hyperbolic simultaneous partial differential equations.

The equations are solved by replacing the former dependent variables, the enthalpy and adsorbate contents of the fluid, by new dependent variables, the characteristic potentials, and by making appropriate assumptions. The characteristic potentials and assumptions are chosen so that the equations are divided into a number of sets of equations. Each set describes the behaviour of only one of the characteristic potentials and is the same as the equations for heat transfer alone except that the characteristic potential replaces temperature as the dependent variable and characteristic specific capacity ratio replaces the matrix/fluid specific heat ratio in the coefficients of the equations. The solution given in the literature for heat regenerators may therefore be used to find a solution for each characteristic potential in turn and hence for the enthalpy and adsorbate contents of the fluid. The resulting solution is an approximation to that of the model because of the assumptions.

The application of this method to air conditioning regenerators is illustrated with a numerical example.

NOMENCLATURE

C_j, $(1 + \mu_j\bar{\sigma}_j)L/(\tau_j v_j)$, parameter describing jth period of sensible heat regenerator [dimensionless];

C_{ij}, $(1 + \mu_j\bar{\gamma}_{ij})L/(\tau_j v_j)$, parameter describing jth period of heat and mass regenerator for ith characteristic potential [dimensionless];

C_{max}, maximum value of $(1 + \mu_j\sigma)L/(\tau_j v_j)$ in matrix [dimensionless];

C_{min}, minimum value of $(1 + \mu_j\sigma)L/(\tau_j v_j)$ in matrix [dimensionless];

E_j, sensible heat regenerator parameters $E_1 = \Lambda_1/(1 + E_4)$, $E_2 = C_2/C_1$, $E_3 = C_1$, $E_4 = \Lambda_1 C_2/(\Lambda_2 C_1)$ [dimensionless];

E_{ij}, heat and mass regenerator parameters for ith characteristic potential $E_{i1} = \Lambda_{i1}/(1 + E_{i4})$, $E_{i2} = C_{i2}/C_{i1}$, $E_{i3} = C_{i1}$, $E_{i4} = \Lambda_{i1}C_{i2}/(\Lambda_{i2}C_{i1})$ [dimensionless];

f, friction factor based on mean wall shear stress [dimensionless];

F_i, ith characteristic potential, $i = 1$ to n [dimensions arbitrary];

J, lumped matrix-fluid film transfer coefficient per unit mass of fluid [s^{-1}];

J_j, value of J during jth period [s^{-1}];

L, depth of matrix in flow direction [m];
n, number of components transferred across matrix–fluid interface [dimensionless];
St, Stanton number [dimensionless];
t, temperature [K];
t', $(t - t_{1,\text{in}})/(t_{2,\text{in}} - t_{1,\text{in}})$, temperature [dimensionless];
$t_{j,\text{in}}$, inlet fluid temperature to period j [K];
$t_{j,\text{out}}$, mean outlet fluid temperature from period j [K];
v, mean velocity of fluid in matrix [m s^{-1}];
v_j, mean velocity of fluid in matrix during period j [m s^{-1}];
w_j, enthalpy, $j = 1$, [Nm kg^{-1}] or sorbate, $j = 2$ to n, [dimensionless] content of the fluid, i.e. enthalpy or jth sorbate mass per unit mass of fluid other than sorbate;
W_j, enthalpy, $j = 1$, [Nm kg^{-1}] or sorbate, $j = 2$ to n, [dimensionless] content of the matrix, i.e. enthalpy or jth sorbate mass per unit mass of matrix other than sorbate;
x, distance from entrance of matrix in fluid flow direction [m];
x', x/L, distance [dimensionless];
γ_i, ith characteristic specific capacity ratio, $i = 1$ to n [dimensionless];
γ_i^i, ith characteristic specific capacity ratio, defined as a function of the ith characteristic potential alone [dimensionless];
$\bar{\gamma}_{ij}$, mean of γ_i along a characteristic, see section 6.1 and equation (12) [dimensionless];
Δp, pressure drop in matrix passages [Nm^{-2}];
η_1, $= (t_{1,\text{out}} - t_{1,\text{in}})/(t_{2,\text{in}} - t_{1,\text{in}})$, temperature effectivity for period one [dimensionless];
η_2, $= (t_{2,\text{out}} - t_{2,\text{in}})/(t_{1,\text{in}} - t_{2,\text{in}})$, temperature effectivity for period two [dimensionless];
η_{ij}, F_i effectivity for period j, defined analogously to temperature effectivity [dimensionless];
Λ_j, $= (1 + \mu_j\bar{\sigma}_j)J_jL/(\mu_j\bar{\sigma}_jv_j)$, parameter describing jth period of sensible heat regenerator [dimensionless];
Λ_{ij}, $= (1 + \mu_j\bar{\gamma}_{ij})J_jL/(\mu_j\bar{\gamma}_{ij}v_j)$, parameter describing jth period of heat and mass regenerator for ith characteristic potential [dimensionless];
μ, matrix mass divided by fluid mass contained in matrix, each mass excluding sorbate [dimensionless];
μ_j, value of μ for jth period [dimensionless];
θ, time from beginning of period [s];
θ', $= \theta/\tau_j$, time from beginning of period [dimensionless];
ρ, density of fluid [kg m^{-3}];
σ, matrix/fluid specific heat ratio [dimensionless];
$\bar{\sigma}$, mean value of σ over a discontinuous temperature wave [dimensionless];
$\bar{\sigma}_j$, mean value of σ between sensible heat regenerator inlet fluid temperatures for jth period [dimensionless];
τ_j, length of jth period [s].

Subscripts used with F_i, t, t', w_j
f, at state of fluid;
m, at, or in equilibrium with, state of matrix.

1. INTRODUCTION

REGENERATIVE heat and mass exchangers, termed regenerators, are used or have been proposed for use in a variety of applications. The mass exchanged between fluid and sorbent matrix is termed sorbate, and the sorption process may be adsorption or absorption.

In chemical engineering, cyclic sorption operations in fixed beds or matrices are important [1] and can be considered as heat and mass regenerators. In a common type, an impurity is removed from a fluid by the sorbent matrix and the matrix is periodically regenerated using the same or a different fluid. The drying of industrial gases with silica gel is an example of this type. The regenerative dehumidifier example considered in section 7 is essentially a silica gel air drier.

In air conditioning, the three types of heat and moisture regenerator used differ in the outlet states obtained for given inlet states. These types are the sensible heat regenerator [2] with high heat and low moisture storage capacity in the matrix, the regenerative dehumidifier [3] with low heat and high moisture storage capacity and the total heat regenerator [4] with high heat and high moisture storage capacity. These devices are usually in the form of a rotating matrix with two axial counter flow streams similar to the Ljungström regenerator used for boiler air preheating.

Sensible heat regenerators are used for heat recovery from exhaust air in air conditioning [2] but total heat regenerators give greater heat recovery in hot humid and cold areas [4]. Regenerative dehumidifiers are used when low humidities are required in air conditioning [3].

Interest in sensible heat regenerators and regenerative dehumidifiers has increased lately as two new air conditioning cooling cycles using them have been proposed. These are the Pennington cycle [5, 6] which is being developed in the U.S.A. by the Gas Developments Corporation and the Dunkle cycle [7] which CSIRO, Australia considered for solar air conditioning some years ago. The development of at least the latter cycle has been retarded by the lack of an adequate theory of regenerative dehumidifier operation.

Analogies are useful in treating transport phenomena. The earliest and best known is Reynolds analogy between heat and momentum transport. This analogy is based on a simple comparison of the form of the equations describing heat transport with the form of those describing momentum transport. In coupled heat and mass transfer, analogies to heat transfer alone are not so obvious. However, if appropriately chosen new dependent variables are introduced to replace enthalpy and sorbate content, the differential equations for coupled linear heat and mass transfer may be reduced to the same number of uncoupled differential equations. Each set of uncoupled differential equations describes the behaviour of one of the new dependent variables and is analogous to the set of equations for heat transfer alone. The mathematical methods of matrix algebra are useful in choosing the new dependent variables especially if more than two components are being transferred. If the original coupled equations are non-linear it is usually necessary to make some assumptions in using this approach.

This method was first used by Henry [8] and has since been used by many other authors, discussed in [9]. Close and Banks [9] have indicated that this method may be used in complicated situations, such as coupled heat and mass transfer occurring during fluid flow through a sorbent matrix involving diffusion in the matrix, diffusion in the fluid along the flow direction and transfer across the fluid–matrix boundary.

The heat and mass regenerator model (section 2) and analysis (section 5) used below is similar to that used by Banks *et al.* [10], who applied the analogy method to predict the response of fluid flowing through a sorbent matrix to a step change in inlet state, when heat and a single sorbate were transferred at rates determined by film transfer coefficients. The new dependent variables introduced are called the characteristic potentials F_i and are analogous to temperature t. Associated with the F_i are characteristic specific capacity ratios γ_i which are analogous to the matrix/fluid specific heat ratio σ. The properties of F_i and γ_i have been discussed in detail by Banks [11] for the equilibrium or infinite transfer coefficient case of heat and single sorbate transfer. F_i and γ_i correspond to the characteristic parameters and directions, $\omega_{(k)}$ and $\sigma_{(k)}$, used by Rhee and Amundson [12] in describing equilibrium heat and multi-sorbate transfer. An important assumption in the analysis below is that the film transfer coefficients for heat and all sorbates are equal, which for [10] implied that the Lewis number was unity. The Lewis number is close to unity for air/water-vapour mixtures [13]. The assumption of equal film transfer coefficients will not be valid for all fluid mixtures but it is used here as a first approximation.

The enthalpy and sorbate contents of the matrix, W_j, are usually non-linear functions of the enthalpy and sorbate contents of fluid in equilibrium with the matrix, w_{jm}. This non-linearity requires that two assumptions (sections 5.2.1 and 5.2.2) be made in transforming the coupled equations to uncoupled form. If F_i and γ_i are chosen appropriately, these two assumptions result in negligible error for reasons given in sections 5.2.1 and 5.2.2. The appropriate choice of F_i and γ_i for two stream counter flow regenerators with two transferred components is discussed in section 6.

This non-linearity also results in uncoupled differential equations, which are analogous to heat transfer alone equations with matrix/fluid specific heat ratio dependent on temperature. Solutions to this heat transfer alone problem are not given in the literature but the literature solutions for constant specific heat ratio may be used to provide an approximate solution as described in section 3 for two period or stream counter flow regenerators. The wave theory solutions for heat transfer alone in section 4 provide theoretical justification for this approximate solution. It can be seen from section 4 that the approximate solution predicts the mean outlet temperature over a period better than it does the variation in outlet temperature. In air conditioning the mean outlet state is more important than the variation in outlet state.

The two assumptions and the approximate solution mentioned above are unnecessary if the enthalpy and sorbate contents of the matrix are linear functions of the enthalpy and sorbate contents of fluid in equilibrium with the matrix, or if the inlet states are sufficiently close for the functions to be considered linear. For most practical problems the separation of the inlet states requires the use of the assumptions and the approximate solution. This is true of the examples of air conditioning regenerator design given in section 7 to clarify the application of the theory presented in the preceding sections.

Previous mathematical methods of predicting sensible heat regenerator performance have been reviewed by Dunkle and Maclaine-cross [2]. These methods neglect the effect of sorbate storage in the matrix on heat and mass transfer performance, but this effect is included by the authors' method. The authors have not found any literature on mathematically predicting total heat regenerator performance.

Prior attempts at mathematically predicting the performance of regenerative dehumidifiers have been directed at solving the equations for a step change in inlet state of fluid flowing through a porous medium [14–16]. This approach gives satisfactory prediction only if the entire bed is in equilibrium with the inlet fluid at the end of each period. Since this condition results in reduced dehumidifying efficiency it is not often satisfied in practice. The theory presented in this paper does not require the above condition.

2. THE SYSTEM MODEL AND EQUATIONS

Heat and mass transfer in regenerators takes place due to the sorbent matrix exchanging heat and mass alternatively with each of the fluid streams. Heat and mass transfer also takes place due to portions of the fluid streams bypassing the regenerator. This effect may be treated as for sensible heat regenerators [2] and is not discussed here. In chemical engineering regenerators the matrix is usualy stationary and the fluid streams are alternatively passed through the matrix using valving. In most air conditioning regenerators the matrix is mounted in a frame which is rotated in a housing. The housing has seals to separate the air streams and ensure that they pass through the matrix. The matrix and frame usually form a wheel (Fig. 1) with axial flow but could be a drum with radial flow.

The equations describing heat and mass transfer in a small element of the matrix in the flow direction (Fig. 1) will be considered below.

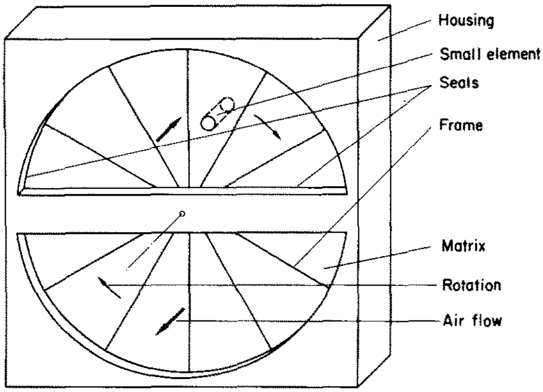

FIG. 1. Two stream counter flow air conditioning regenerator with a small element marked out as discussed in Section 2.

The matrix may be considered to be made up of a number of similar small elements each subject to the same boundary conditions and with the same time mean outlet states. The time mean outlet state of one such small element during a given period is the mean outlet state of the whole regenerator for the flow stream corresponding to the given period.

The following model (Fig. 2) of an element will be used. The enthalpy and sorbate contents of the matrix, W_j, fluid, w_{jf}, and fluid in equilibrium with the matrix, w_{jm}, vary only with distance in the flow direction and time. Sorption hysteresis is neglected so that the W_j are single-valued functions of the w_{jm}. The mass ratio μ and velocity v are constant. Diffusion and dispersion in the fluid flow direction and transfer flux coupling are neglected.

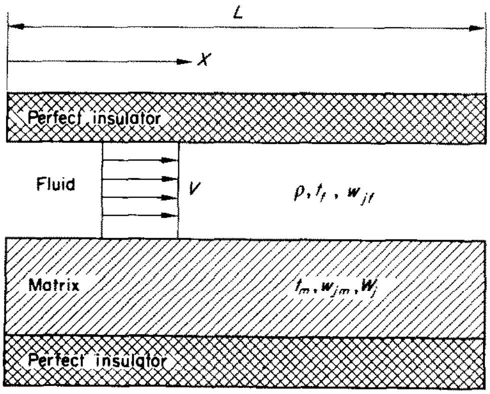

FIG. 2. Model of small element of matrix.

Transfer between the fluid and matrix is assumed to be described by a single film transfer coefficient J. This assumption is satisfactory in air conditioning where the Lewis number is close to unity, [13], and in some chemical engineering problems where a number of similar sorbates are being transferred.

The conservation equations are, following [10] and justification [11],

$$\frac{\partial w_{jf}}{\partial \theta} + v\frac{\partial w_{jf}}{\partial x} + \mu\frac{\partial W_j}{\partial \theta} = 0, \quad j = 1 \text{ to } n. \qquad (1)$$

The transfer rate equations may be written, compare [10],

$$\mu\frac{\partial W_j}{\partial \theta} + J(w_{jm} - w_{jf}) = 0, \quad j = 1 \text{ to } n. \qquad (2)$$

These equations are solved with the equilibrium relationships, compare [10],

$$W_j = W_j(w_{km}), \; k = 1 \text{ to } n, \quad j = 1 \text{ to } n. \qquad (3)$$

3. GENERAL THEORY OF HEAT TRANSFER ALONE

If heat alone is transferred equations (1) and (3) for $j = 1$ become

$$\frac{\partial t_f}{\partial \theta} + v\frac{\partial t_f}{\partial x} + \mu\sigma\frac{\partial t_m}{\partial \theta} = 0 \qquad (4)$$

and equations (2) and (3) for $j = 1$ become, assuming that the specific heat of the fluid is constant,

$$\mu\sigma\frac{\partial t_m}{\partial \theta} + J(t_m - t_f) = 0. \qquad (5)$$

Equations (4) and (5) are conventionally solved, references given in [2], by assuming constant specific heat ratio σ and negligible fluid heat storage term $\partial t_f/\partial\theta$ in equation (4). In the following three paragraphs it will be indicated why these two conventional approximations are unsatisfactory for the purposes of this paper and two alternative approximations will be proposed and justified.

The assumption of constant σ is satisfactory for sensible heat regenerators, but the use of the analogy method for regenerative dehumidifiers and total heat regenerators requires consideration of large variations in σ, which are almost 5:1 in the examples in section 7. It is shown in section 4 that replacing σ by its mean value between the two inlet temperatures for a given period $\bar{\sigma}_j$, then using constant σ solutions is a good approximation for the limiting case of high J. For finite J the variation of matrix temperature and hence σ is less, so the approximation is expected to improve. Furthermore this approximation gives the correct result if σ is constant, so that it may be used for all cases.

The assumption of negligible $\partial t_f/\partial\theta$ in equation (4) is satisfactory if $\mu\sigma \gg 1$, but the application of the analogy method to air conditioning sensible heat regenerators requires consideration of low values of $\mu\sigma$ and for the sensible heat regenerator example in section 7, $\mu\sigma = 0$. Temperature distributions assuming constant σ and negligible $\partial t_f/\partial\theta$ in equation (4) have been given by Hausen [17] and Mondt [18] for counter flow regenerators and by Johnson [19] and Kays and London [20], Chapter 3, for a step change in inlet temperature of fluid flowing through a porous medium. Hausen [17] discussed the approximation $\partial t_f/\partial\theta \simeq \partial t_m/\partial\theta$ in detail for symmetric counter flow regenerators, and applied it to determine the effect of solid diffusion in the matrix. The temperature distributions and Hausen's discussion show that $\partial t_m/\partial\theta$ always lies between $\partial t_f/\partial\theta$ and zero, and, except near the fluid inlet at the start of a period or immediately after the step change, $\partial t_f/\partial\theta \simeq \partial t_m/\partial\theta$. This suggests that replacing $\partial t_f/\partial\theta$ in equation (4) by $\partial t_m/\partial\theta$ is a better approximation than replacing it by zero.

In practical cases, when $\mu\sigma$ is small $C_{max} \ll 1$ also, and section 4.2 shows that the authors' approximation gives exact values of effectivity. If σ is constant and a transformed time coordinate $\theta - x/v$ is introduced to eliminate the fluid storage term $\partial t_f/\partial\theta$ from equation (4) an exact solution is obtained for the step change case [19, 20]. Unfortunately this transformation does not lead to a general solution for the counter flow regenerator due to difficulties with boundary conditions, but for the special case of a regenerator with $C_{max} \ll 1$ the exact step change solution is applicable. Figure 3, which is a

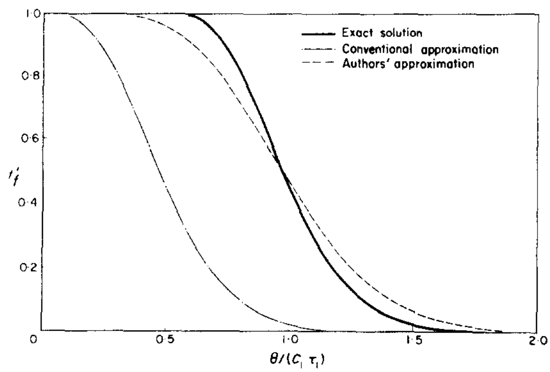

FIG. 3. Dimensionless outlet fluid temperature t'_f as a function of the dimensionless time $\theta/C_1\tau_1$ for period one of a regenerator with $\Lambda_1 = 20$, $\mu\sigma = 1$ and $C_{max} \ll 1$. The exact solution is compared with the conventional approximation ($\partial t_f/\partial\theta = 0$) and the authors' approximation ($\partial t_f/\partial\theta = \partial t_m/\partial\theta$).

diagram of dimensionless outlet temperature against dimensionless time for $\mu\sigma = 1$ and $\Lambda = 20$, shows that the authors' approximation is much closer to the exact solution than the conventional approximation. The authors' approximation may be used for all practical cases because when the error from the conventional approximation is large it gives superior results (this paragraph), and when the error from the conventional approximation is small there is good reason to suspect that the error from the authors' approximation is smaller (previous paragraph).

The two new approximations discussed above will be used throughout below so that a single set of design equations may be applied to all cases. With the two approximations and introducing the dimensionless co-ordinates t'_f, t'_m, x' and θ' and the dimensionless parameters C_j, Λ_j

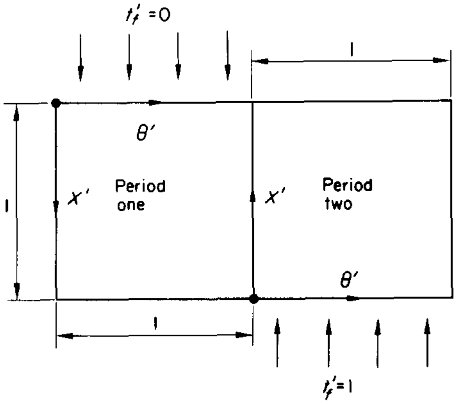

FIG. 4. Dimensionless boundary conditions for two period counter flow regenerator.

equations (4) and (5) become

$$\frac{\partial t_f'}{\partial x'} + C_j \frac{\partial t_m'}{\partial \theta'} = 0, \qquad j = 1, 2 \tag{6}$$

$$C_j \frac{\partial t_m'}{\partial \theta'} + \Lambda_j(t_m' - t_f') = 0, \quad j = 1, 2 \tag{7}$$

where the subscript j indicates the flow period. Period one is chosen so that $C_1 \geqslant C_2$. The dimensionless parameters C_j, Λ_j are constants for each period.

Since the boundary conditions for equations (6) and (7) (Fig. 4) are the same for all two period counter flow regenerators, the effectivity or dimensionless mean outlet temperature for period one, η_1, is a function of the four

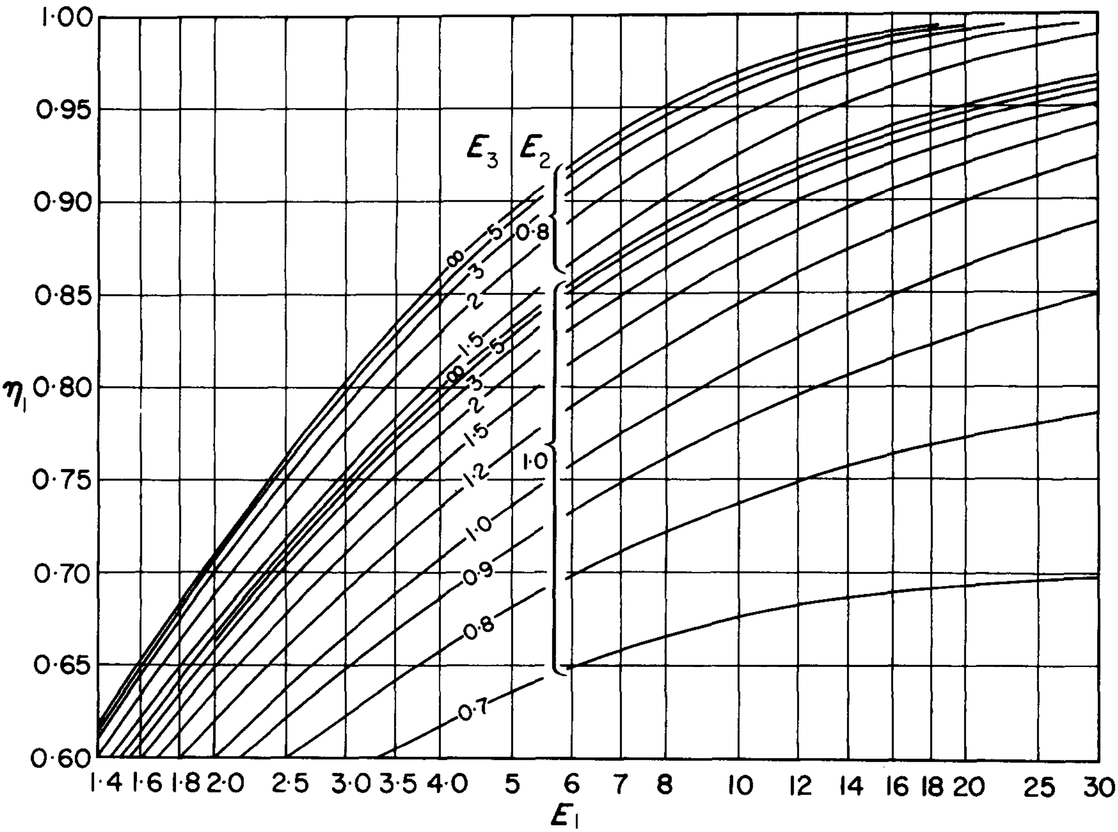

FIG. 5. Effectivity η_1 for $E_2 = 1{\cdot}0$ and $0{\cdot}8$ and $E_4 = 1$.

dimensionless parameters C_j, Λ_j. In the literature η_1 is usually presented as a function of the four dimensionless parameters E_j where $E_1 = \Lambda_1/(1 + E_4)$, $E_2 = C_2/C_1$, $E_3 = C_1$ and $E_4 = \Lambda_1 C_2/(\Lambda_2 C_1)$. The symbols used by Kays and London [20], Chapter 2, for E_1, E_2, E_3 and E_4 are $N_{tu, 0}$, C_{min}/C_{max}, C_r/C_{min} and $(hA)^*$ respectively. speed of rotation. The effect of E_4 is small and the values of effectivity for $E_4 = 1$ are a good approximation to those for $0{\cdot}25 < E_4 < 4$ if $E_3 > 2$. Figures 5 and 6 give η_1 as a function of the E_j and were prepared with an electronic digital computer using an improved version of Lambertson's method [21].

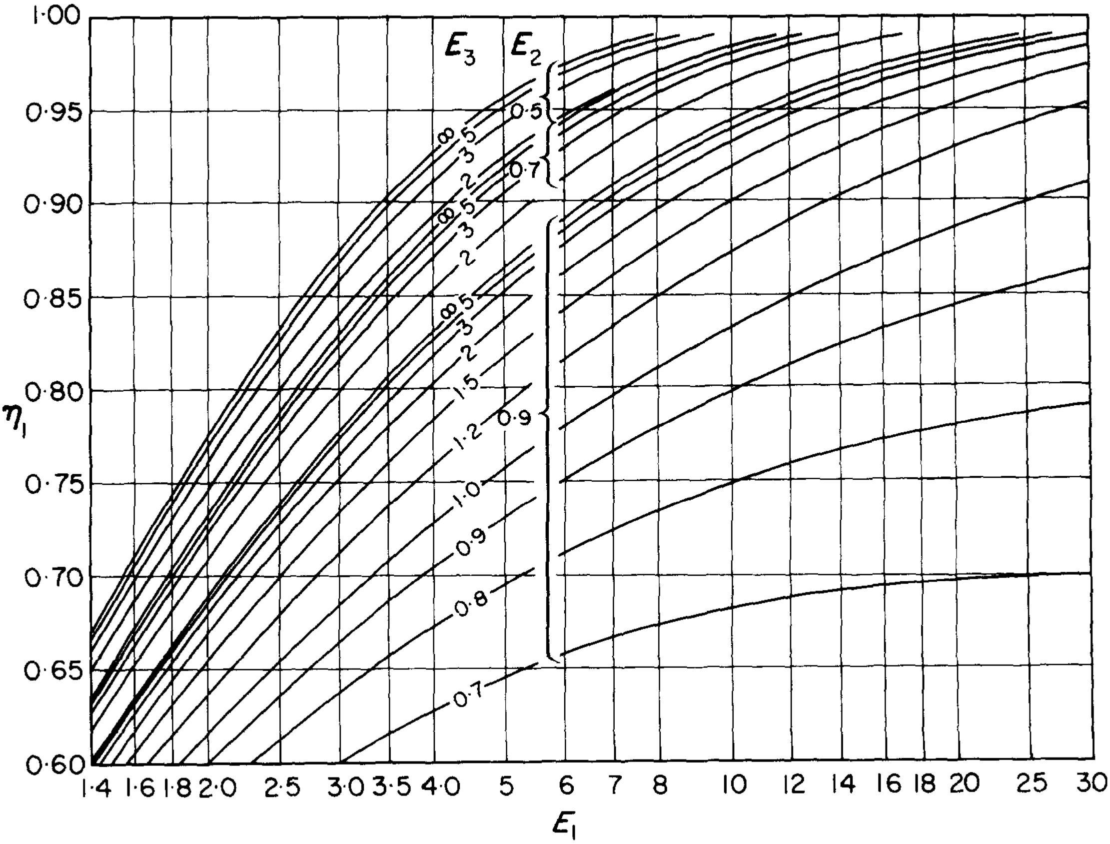

FIG. 6. Effectivity η_1 for $E_2 = 0{\cdot}9$, $0{\cdot}7$ and $0{\cdot}5$ and $E_4 = 1$.

The effectivity for period two, η_2, may be obtained from an overall energy balance giving $\eta_2 = \eta_1 E_2$.

The number of transfer units E_1 is the most important parameter determining the effectivity η_1. The fluid stream capacity rate ratio E_2 gives the effect of unbalanced flows on η_1 and the matrix/fluid capacity rate ratio E_3 the effect of

4. THE WAVE THEORY OF HEAT TRANSFER ALONE

The limiting case of large E_3 has proved very useful in developing the general theory of heat transfer in regenerators, [20] p. 29. There is another limiting case which has not been widely used in the literature but will be found particularly useful here. This is the limiting case of large E_1 or infinite transfer coefficients. In this case, from equation (5) $t_f = t_m$ and equation (4) becomes a first order non-linear wave equation, discussed in [11, 12] and its solution for the two period counter flow regenerator boundary conditions will be described below.

The solution is a system of temperature waves which satisfy the boundary conditions and equation (4) with $t_f = t_m$, and give a single value of temperature t_f for all values of x and θ. Widening, narrowing or discontinuous temperature waves may occur. The velocity of a small element in a widening or narrowing wave may be shown from equation (4) to be $v/(1 + \mu\sigma)$ and the velocity of a discontinuous wave $v/(1 + \mu\bar{\sigma})$.

A dimensionless wave diagram will be used below to illustrate four wave theory solutions of special interest, which are discussed in terms of the dimensionless parameters, C_j. The coordinates of this diagram are dimensionless length x' and dimensionless time θ' as for Fig. 4. The paths of representative wave elements are shown as light lines and the paths of discontinuous waves as heavy lines.

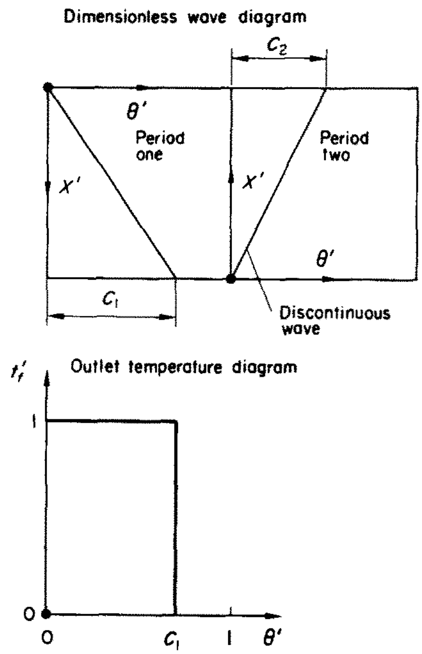

FIG. 7. Dimensionless wave diagram and period one outlet temperature diagram for constant specific heat ratio and $C_1 < 1$.

4.1 *Constant specific heat ratio*

If $C_1 < 1$ the discontinuous waves on the dimensionless wave diagram in Fig. 7 are the solution, and from the outlet temperature diagram $\eta_1 = C_1$. If $C_1 \geqslant 1$ it can be shown similarly that $\eta_1 = 1$.

4.2 *Low rotational speed*

If the rotational speed is so low and hence flow period so long that $C_{max} < 1$, then at the start of period one the matrix is at the inlet temperature to period two, and at the end of period one it is at the inlet temperature to period one, (Fig. 8) From an energy balance on period one, $\eta_1 = C_1$. This method may also be applied to the case of finite J if $C_{max} \ll 1$ to show that $\eta_1 = C_1$. In fact the authors' digital computer solutions show that if $E_1 \geqslant 4{\cdot}5$, $E_2 \leqslant 1$, $E_3 \leqslant 0{\cdot}4$ and $E_4 = 1$ then $0 < C_1 - \eta_1 < 0{\cdot}001$.

4.3 *High rotational speed*

Consider rotational speeds so high that $C_{min} > 1$. In the cyclic wave pattern in the

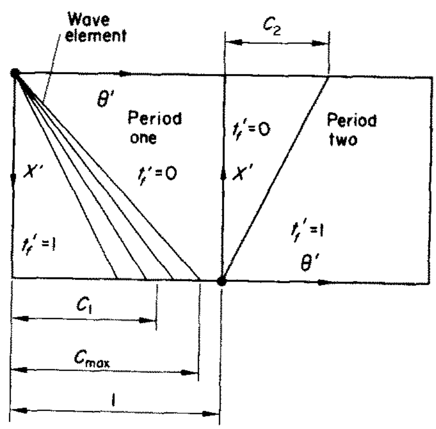

FIG. 8. Dimensionless wave diagram for $C_{max} < 1$.

matrix, there are three groups of wave elements, firstly elements which always remain in the matrix, secondly elements which enter the matrix at the beginning of period one and thirdly elements which enter the matrix at the beginning of period two.

It will now be shown that no element from any of these three groups can pass out of the matrix during period one. For the first group this is obvious. Elements of the second group cannot pass out during period one because their maximum dimensionless wave velocity $1/C_{min}$ is less than one. Elements of the third group cannot pass out during period one since their mean dimensionless wave velocity is less during period one than during period two because $C_1 > C_2$.

Since no wave elements pass out during period one, the outlet temperature from period one is the same as the inlet temperature to period two, so that the effectivity $\eta_1 = 1$ if $C_{min} > 1$.

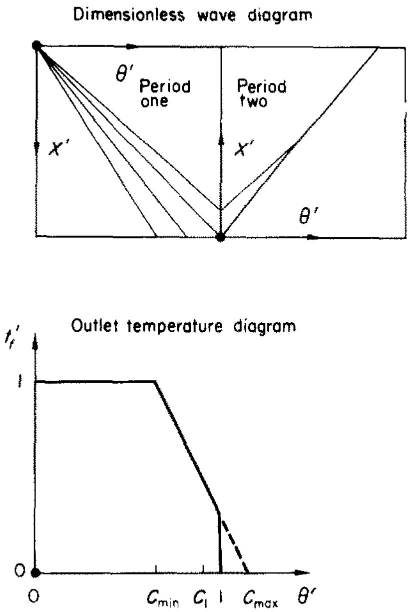

FIG. 9. Dimensionless wave diagram and period one outlet temperature diagram for σ linearly dependent on temperature and $C_{min} \leqslant 1 \leqslant C_{max}$.

4.4 *Specific heat ratio linearly dependent on temperature*

If σ is linearly dependent on temperature, $(1 + \mu\sigma)$ is linearly dependent on temperature. The maxima and minima of $(1 + \mu\sigma)$ and hence C_{max} and C_{min} occur at the two inlet temperatures. It will be assumed that the solution has a widening wave during period one (Fig. 8) as this gives the greatest error in η_1 due to replacing σ by $\bar{\sigma}$ in equations (4) and (5).

From section 4.2, if $C_{max} < 1$, $\eta_1 = C_1$. If $C_{min} \leqslant 1 \leqslant C_{max}$, from the outlet temperature diagram (Fig. 9)

$$\eta_1 = 1 - \frac{(1 + C_{max}/C_{min} - 2C_1)^2}{4C_1[(C_{max}/C_{min})^2 - 1]}.$$

From section 4.3 if $C_{min} > 1$, $\eta_1 = 1$.

This solution for η_1 which has been plotted in Fig. 10 shows that, even for variations in σ as large as 5:1, the error due to replacing σ by $\bar{\sigma}$ in equation (4) is small except near $C_1 = 1$. From sections 4.2 and 4.3 this will be true even if the variation in σ is non-linear. In using the heat transfer only solution in the analogy method described below, variations of σ as large as 5:1 are encountered but C_1 is far from one, as illustrated by the examples in section 7.

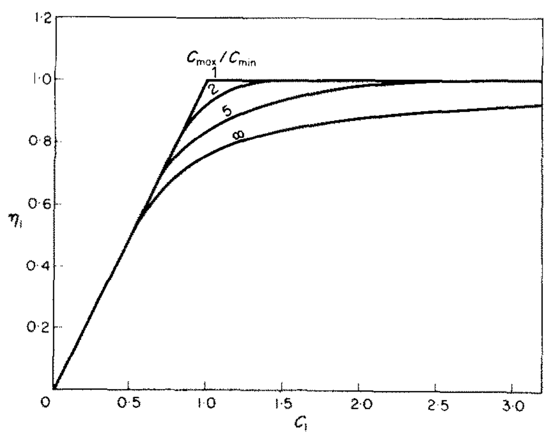

FIG. 10. Effectivity η_1, for infinite film transfer coefficient and σ linearly dependent on temperature, as a function of C_1 and C_{max}/C_{min}.

5. THE ANALOGY WITH HEAT TRANSFER ALONE

To transform equations (1)–(3) into a form analogous to equations (4) and (5) new variables must be introduced. The present dependent variables, the enthalpy and sorbate content of the fluid, w_j, will be replaced by new dependent variables, the characteristic potentials, F_i. These potentials are analogous to temperature in heat transfer alone. Also characteristic specific capacity ratios, γ_i, which are analogous to σ, will be introduced.

Banks *et al.* [10] used F_i and γ_i in describing coupled heat and single sorbate transfer with finite film transfer coefficients between fluid and matrix. The model and analysis used here differs from Banks *et al.* in that the Lewis number and its mass transfer equivalents are assumed unity initially, the number of components transferred is arbitrary, and enthalpy is used in describing fluid and matrix states instead of temperature. This last difference simplifies the equations and is consistent with the use of Mollier type psychrometric charts in air conditioning.

5.1 *The new variables* (F_i, γ_i)

Let γ_i be the ith root of the nth order polynomial equation in determinant form

$$\det\left[\frac{\partial W_j}{\partial w_k} - \delta_{jk}\gamma\right] = 0 \qquad (8)$$

and let $\partial F_i/\partial w_j$, $j = 1$ to n, be the solutions of the simultaneous linear algebraic equations

$$\sum_{j=1}^{j=n} \frac{\partial F_i}{\partial w_j}\frac{\partial W_j}{\partial w_k} = \frac{\partial F_i}{\partial w_k}\gamma_i, \quad k = 1 \text{ to } n. \qquad (9)$$

In matrix algebra terms [22], γ_i is the ith characteristic value and $[\partial F_i/\partial w_j]$ the corresponding left characteristic vector of the matrix $[\partial W_j/\partial w_k]$. The vectors $[\partial F_i/\partial w_j]$ define the directions of the normals to the surfaces of constant F_i and hence the surfaces themselves in the n-dimensional w_j space. Since the vectors $[\partial F_i/\partial w_j]$ are or may be chosen to be linearly independent and the F_i are functions of the w_j, the w_j may be expressed as single valued functions of the F_i. This result is intuitively obvious and is proved in texts on calculus [23]. Hence the F_i may be used to specify the state of the fluid in place of the w_j.

For the case of two w_j dimensions or two components transferred, the surfaces of constant F_i become two sets of lines which may be plotted on a chart with w_1 and w_2 as co-ordinates. Lines of constant γ_i may also be plotted in this fashion. If the fluid is air, w_1 enthalpy content and w_2 moisture content, then constant F_i and γ_i lines may be plotted on Mollier type psychrometric charts.

Close and Banks [24] have prepared such charts for a simplified silica-gel/water/air system. Figure 11 shows lines of constant F_i and Fig. 12 γ_i on Mollier type psychrometric charts. The F_1 lines lie close to adiabatic saturation lines and the F_2 lines to relative humidity lines. γ_1 lines are similar to F_1 lines and γ_2 to F_2 lines. γ_2 is of the order of a hundred times γ_1.

5.2 *The* F_i *equations*

Making the two assumptions discussed in sections 5.2.1 and 5.2.2 it is shown in sections 5.2.3 and 5.2.4 that the F_i satisfy the following equations

$$\frac{\partial F_{if}}{\partial \theta} + v\frac{\partial F_{if}}{\partial x} + \mu\gamma_i^i\frac{\partial F_{im}}{\partial \theta} = 0, \quad i = 1 \text{ to } n \qquad (10)$$

$$\mu\gamma_i^i\frac{\partial F_{im}}{\partial \theta} + J(F_{im} - F_{if}) = 0, \quad i = 1 \text{ to } n \qquad (11)$$

where the subscript f refers to the fluid state and the subscript m the matrix state.

5.2.1. Assume $\partial F_i/\partial w_j$, $i = 1$ to n, $j = 1$ to n, are constant on a path between the fluid and matrix states. If the fluid is in equilibrium with the matrix, this assumption is exact. If the values of F_i on the surfaces of constant F_i are chosen such that the F_i are as nearly as possible linear functions of the w_j (section 6), and the transfer coefficient J is high so that fluid and matrix states are close, this assumption causes negligible error. This will be the case in most practical problems and in the examples considered in section 7.

5.2.2. Assume $\gamma_{im} = \gamma_i^i$. Each γ_{im} is a function of all the F_{im}, $i = 1$ to n, but γ_i^i is a function of F_{im} only. If the function γ_i^i is obtained by substituting approximate values of $F_{jm}, j \neq i$, in γ_{im} (section 6), γ_i^i will be close to γ_{im}. In the examples considered in section 7, γ_{im} depends mainly on F_{im}, as can be seen from Figs. 11 and 12.

5.2.3. *Proof of equation (10).*

$$\text{L.H.S.} = \sum_{j=1}^{j=n} \frac{\partial F_{if}}{\partial w_{jf}}\left(\frac{\partial w_{jf}}{\partial \theta} + v\frac{\partial w_{jf}}{\partial x}\right) + \mu \sum_{k=1}^{k=n} \gamma_{im}\frac{\partial F_{im}}{\partial w_{km}}\frac{\partial w_{km}}{\partial \theta} \quad \text{[expanding derivatives and 5.2.2]}$$

$$= \sum_{j=1}^{j=n} \frac{\partial F_{if}}{\partial w_{jf}}\left(\frac{\partial w_{jf}}{\partial \theta} + v\frac{\partial w_{jf}}{\partial x}\right) + \mu \sum_{k=1}^{k=n}\sum_{j=1}^{j=n} \frac{\partial F_{im}}{\partial w_{jm}}\frac{\partial W_j}{\partial w_{km}}\frac{\partial w_{km}}{\partial \theta} \quad \text{[equation (9)]}$$

$$= \sum_{j=1}^{j=n} \frac{\partial F_{if}}{\partial w_{jf}}\left(\frac{\partial w_{jf}}{\partial \theta} + v\frac{\partial w_{jf}}{\partial x} + \mu\frac{\partial W_j}{\partial \theta}\right) \quad \text{[contracting derivatives and 5.2.1]}$$

$$= \text{R.H.S.} \quad \text{[equation (1)]}$$

5.2.4. *Proof of equation* (11).

$$\text{L.H.S.} = \mu \sum_{k=1}^{k=n} \gamma_{im}\frac{\partial F_{im}}{\partial w_{km}}\frac{\partial w_{km}}{\partial \theta} + J(F_{im} - F_{if}) \quad \text{[expanding derivatives and 5.2.2]}$$

$$= \mu \sum_{k=1}^{k=n}\sum_{j=1}^{j=n} \frac{\partial F_{im}}{\partial w_{jm}}\frac{\partial W_j}{\partial w_{km}}\frac{\partial w_{km}}{\partial \theta} + J\sum_{j=1}^{j=n}\frac{\partial F_{im}}{\partial w_{jm}}(w_{jm} - w_{jf}) \quad \text{[equation (9) and 5.2.1]}$$

$$= \sum_{j=1}^{j=n} \frac{\partial F_{im}}{\partial w_{jm}}\left[\mu\frac{\partial W_j}{\partial \theta} + J(w_{jm} - w_{jf})\right] \quad \text{[contracting derivatives]}$$

$$= \text{R.H.S.} \quad \text{[equation (2)]}$$

5.3 *General method of applying the analogy*

The pair of equations for $i = l$ describe changes in F_l alone. Regenerator initial and boundary conditions in terms of w_{jf}, w_{jm}, $j = 1$ to n, may be expressed in terms of F_{if}, F_{im}, $i = 1$ to n. Hence each pair of equations (10) and (11) may be solved independently of the other pairs. The equations in each pair have the same form as those for heat transfer alone [equations (4) and (5)]. Hence if the $\bar{\sigma}_j$ are replaced by mean values of the γ_i^i in the dimensionless parameters E_j, the effectivity η_j calculated using the methods described in sections 3 and 4 is the F_i effectivity η_{ij}. The η_{ij} may be used to obtain the F_i for the mean outlet states and hence the w_j for these states.

6. CHOICE OF F_i AND γ_i^i FOR REGENERATOR DESIGN

Methods of choosing F_i and γ_i^i for a two period counter flow regenerator with two components transferred, such that assumptions 5.2.1 and 5.2.2 are closely satisfied, will be given below for three rotational speeds or period lengths. The

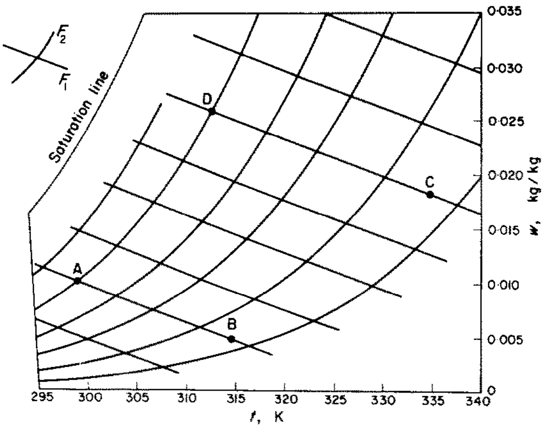

FIG. 11. F_i chart for a simplified silica-gel/water/air system.

choice of F_i and γ_i^i for more complicated cases may be made using generalizations of the methods below. Energy and mass balances on the two inlet and two mean outlet states may be used to check the accuracy of the method.

The values of F_i on the surfaces of constant F_i are required in the general method (section 5.3) to determine the mean outlet w_j from the η_{ij}. It is equally satisfactory, however, to have a method of determining the mean outlet w_j from the η_{ij} without using values of F_i. Such a method is given in section 6.1. This method may be justified in the low speed case and partially justified for the medium speed case by using

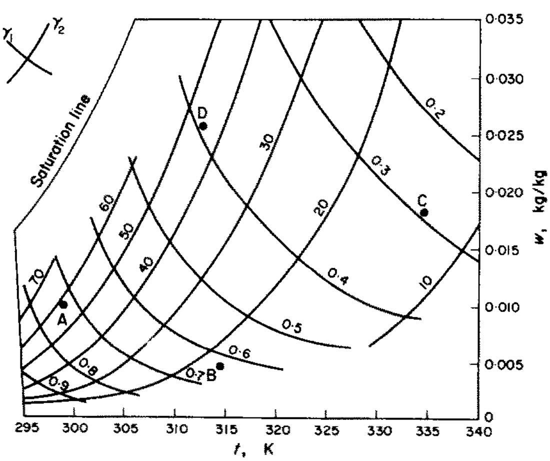

FIG. 12. γ_i chart for a simplified silica-gel/water/air system.

energy and mass balances on period one. In any case the choice of F_i implied using this method satisfies the condition for a reasonable choice of F_i given in section 5.2.1.

γ_i^i may be determined as a function of F_i using sections 5.1 and 5.2.2 but the general method (section 5.3) requires values of the mean γ_i^i, $\bar{\gamma}_{ij}$, which replaces $\bar{\sigma}$ in the C_j, Λ_j. The $\bar{\gamma}_{ij}$ for the low speed case (section 6.1) are chosen using energy and mass balances, and for the medium (section 6.2) and high speed (section 6.3) cases using sections 4 and 5.2.2.

The simplified silica-gel/water/air system, derived in [24], in which heat and moisture are transferred will be used to illustrate the discussion. The inlet state for period one is point A on the F_i and γ_i charts for this system (Figs. 11 and 12) and the inlet state for period two is point C. Point B is the intersection of the F_2 line through point C with the F_1 line through point A. Point D is the intersection of the F_2 line through point A with the F_1 line through point C. Period one was chosen so that $C_{21} \geqslant C_{22}$ below.

6.1 *Low rotational speeds*

Low rotational speeds are defined as occurring when $C_{ij} \ll 1$. This means that at the start of period one the matrix is at the inlet state to period two (point C) and at the end of period one it is at the inlet state to period one (point A). An energy and mass balance may thus be used to calculate the mean outlet state for period one exactly, and similarly for period two. The method below gives a close approximation to this exact solution.

$\bar{\gamma}_{11}$ is the mean of γ_1 with respect to water content w_2 along the length of F_2 line between B and C so that

$$\bar{\gamma}_{11} = \frac{\int_{w_{2B}}^{w_{2C}} \gamma_1 \, dw_2}{w_{2C} - w_{2B}}. \qquad (12)$$

$\bar{\gamma}_{12}$ is a similar mean of γ_1 between D and A, $\bar{\gamma}_{21}$ of γ_2 along the F_1 line between A and B and $\bar{\gamma}_{22}$ of γ_2 between C and D. The integral in equation (12) may be evaluated using Simpson's rule or other approximate methods of integration. The C_{ij} may now be calculated, and from the analogy (section 5) and the appropriate heat transfer alone solution (section 4.2), $\eta_{ij} = C_{ij}$. The mean outlet state for period one is then given by

$$w_j = w_{jA} + \eta_{11}(w_{jC} - w_{jB}) + \eta_{21}(w_{jB} - w_{jA}), \quad j = 1, 2. \tag{13}$$

There are similar equations for period two.

6.2 *Medium rotational speeds*

Medium rotational speeds are defined as occurring when $C_{2j} \gg 1$ and $C_{1j} \ll 1$. Since $C_{2j} \gg 1$, it can be shown using the analogy and the heat transfer alone theory that there is an approximately linear distribution of F_2 through the matrix. Since $C_{1j} \ll 1$, waves in F_1 propagate through the matrix with a velocity which may be obtained from the analogy and section 4. The velocity of the F_1 waves depends on F_2 and hence varies through the matrix resulting in a curved line on the dimensionless F_1 wave diagram (Fig. 13). The approximate matrix states are also marked on the wave diagram.

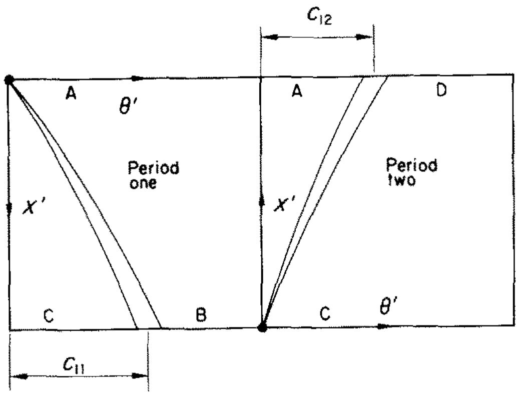

FIG. 13. Dimensionless F_1 wave diagram.

By integration the appropriate value for $\bar{\gamma}_{11}$ is found to be the mean of γ_1 over the area ABCD on the γ_i chart (Fig. 12). Similarly $\bar{\gamma}_{12} = \bar{\gamma}_{11}$. A satisfactory approximation to this mean value for practical purposes is the value of γ_1 half way between the inlet states A and C on the γ_i chart.

For medium speeds, the evaluation of the $\bar{\gamma}_{2j}$ is more complicated than the evaluation of the $\bar{\gamma}_{1j}$. As can be seen from the F_1 wave diagram (Fig. 13), during period one most of the matrix lies on the F_2 line AB but about $C_{11}/2$ lies on F_2 line CD. If C_{11} is very small, it is reasonable to assume that the matrix lies on AB for all period one and to evaluate $\bar{\gamma}_{21}$ as for low speeds. If C_{11} is larger, the effect of the part of the matrix on CD is approximately included if $\bar{\gamma}_{21}$ is evaluated as

$$\bar{\gamma}_{21} = \left(1 - \frac{C_{11}}{2}\right) \frac{\int_{w_{2A}}^{w_{2B}} \gamma_2 \, dw_2}{w_{2B} - w_{2A}} + \frac{C_{11}}{2} \frac{\int_{w_{2C}}^{w_{2D}} \gamma_2 \, dw_2}{w_{2D} - w_{2C}}. \tag{14}$$

$\bar{\gamma}_{22}$ may be evaluated similarly.

Using the values of γ_{ij} obtained above, the E_{ij} may be evaluated and hence the η_{ij}. The mean outlet w_j may be obtained using equation (13) as for low speeds.

6.3 *High rotational speeds*

High rotational speeds are defined as occurring when $C_{ij} \gg 1$. Since $C_{ij} \gg 1$, it can be shown using the analogy that there is an approximately linear distribution of F_1 and F_2 in the matrix. The matrix states lies on a straight line between the two inlet states, A and C. Using section 5.2.2 the appropriate values of $\bar{\gamma}_{ij}$ are the means of γ_i along this line with respect to water content w_2. Since the same line is used for both inlet states $\bar{\gamma}_{i1} = \bar{\gamma}_{i2}$. A useful approximation to these means is to take the values of γ_i at the midpoint of the straight line AC. Using these $\bar{\gamma}_{ij}$ the η_{ij} and mean outlet w_j may be obtained as in section 6.2.

7. EXAMPLES OF REGENERATOR DESIGN

The theory in the preceding sections will be applied below to the design of three air conditioning regenerators. They are a sensible heat regenerator with a matrix of randomly packed glass beads, and a regenerative dehumidifier and a total heat regenerator each with a matrix of randomly packed silica-gel beads. The glass beads are assumed to adsorb negligible moisture, so that their F_1 characteristics are lines of constant air moisture content, F_2 constant temperature and $\gamma_1 = 0$, $\gamma_2 = \sigma$ [11]. Glass and silica gel beads are used for the matrices because the necessary properties data are available (Figs. 11 and 12) and not because they are suitable for any particular application.

For the sensible heat regenerator and regenerative dehumidifier the η_{1j} should be very small and the η_{2j} close to one. This is obtained by choosing a matrix with $\gamma_2 \gg \gamma_1$. If a medium rotational speed is then chosen so that the $C_{1j} \ll 1$ and $C_{2j} \gg 1$, η_{1j} will be very small and, if the E_{i1} are large, the η_{2j} will be close to one. For the total heat regenerator the η_{ij} should all be close to one. If $\gamma_2 > \gamma_1$ choosing $C_{1j} > 1$ will ensure that $C_{2j} > 1$ and so the η_{ij} will all be close to one if the E_{i1} are large.

All three examples of regenerator design will have the same shape and length measurements. The two air streams are balanced and counter flow and the regenerators are symmetric. The design problem which will be solved below is given η_{21}, E_{13}, Δp, μ, ρ, L, St/f, charts of F_i and γ_i (Figs. 11 and 12), the inlet air states, and charts or tables of η_1 (Figs. 5 and 6), determine v_1, τ_1 and the two mean outlet states.

The following expressions for balanced symmetric air conditioning regenerators may be derived from the definitions and will be used below.

$$E_{i1} = \frac{St\Delta p 2(1 + \mu\bar{\gamma}_{i1})}{f\rho v_1^2 \mu(\bar{\gamma}_{i1} + \bar{\gamma}_{i2})}, \qquad i = 1, 2 \quad (15)$$

$$E_{i2} = \frac{1 + \mu\gamma_{i2}}{1 + \mu\gamma_{i1}}, \qquad i = 1, 2 \quad (16)$$

$$E_{i3} = (1 + \mu\bar{\gamma}_{i1})\frac{L}{\tau_1 v_1}, \qquad i = 1, 2. \quad (17)$$

Corresponding expressions may be derived for unbalanced unsymmetric regenerators.

7.1 *The method of solution*

The method of solution for the regenerative dehumidifier consists of the following steps. The steps are similar for the sensible and total heat regenerators.

7.1.1. Calculate $\bar{\gamma}_{ij}$ using section 6.2 and γ_i chart, like Fig. 12.

7.1.2. From equation (17) determine E_{23}, $E_{23} = E_{13}(1 + \mu\bar{\gamma}_{21})/(1 + \mu\bar{\gamma}_{11})$.

7.1.3. From equation (16) determine E_{12} and E_{22}.

7.1.4. Using Figs. 5 and 6 determine E_{21} from η_{21}, E_{22} and E_{23}.

7.1.5. Determine v_1 from equation (15) with $i = 2$.

7.1.6. Calculate τ_1 from equation (17) with $i = 2$.

7.1.7. From the analogy and section 4.2, $\eta_{11} = E_{13}$.

7.1.8. Calculate η_{i2} from $\eta_{i2} = E_{i2}\eta_{i1}$, $i = 1, 2$.

7.1.9. Determine outlet states on F_i chart from the inlet states, η_{ij} and equation (13).

7.1.10. Check overall conservation of heat and moisture by checking that the lines on the F_i chart joining corresponding inlet and outlet states are parallel and equal in length. Discrepancies are expected due to the approximate nature of the solution obtained by the above method, the accuracy of which is discussed in section 8.

7.2 *Numerical data and results*

The numerical data and results for the three examples are given below and in Table 1. The inlet states chosen were temperature t 299·4 K and moisture content w_2 0·0101 kg/kg for period one (point A on Figs. 11 and 12) and 334·2 K and 0·0182 kg/kg for period two (point C on Figs. 11 and 12). The density of air at the mean of the two inlet states $\rho = 1{\cdot}105$ kg m^{-3}. For both matrices $St/f = 0{\cdot}06$, $\Delta p = 100$ N m^{-2} and $L = 50$ mm.

The difference in outlet states for the same inlet state which can be seen in Fig. 14 is due to the different matrices and rotational speeds used.

8. ACCURACY OF THE ANALOGY METHOD

The analogy method for heat and mass regenerators gives approximate solutions to the model described in section 2, because of the assumptions in sections 3, 5.2.1 and 5.2.2. For the step change case, Banks *et al.* [10] attributed most error in instantaneous outlet state to the assumption of constant specific heat ratio in the heat transfer alone solution used in the analogy method. For the regenerator, section 4 suggests that there is little error in mean outlet state due to this cause, unless the C_{ij} are close to one, when Fig. 10 would give an estimate of the error. The accuracy of the analogy method remains to be determined exactly by comparing with accurate finite difference solutions of the basic equations of the model.

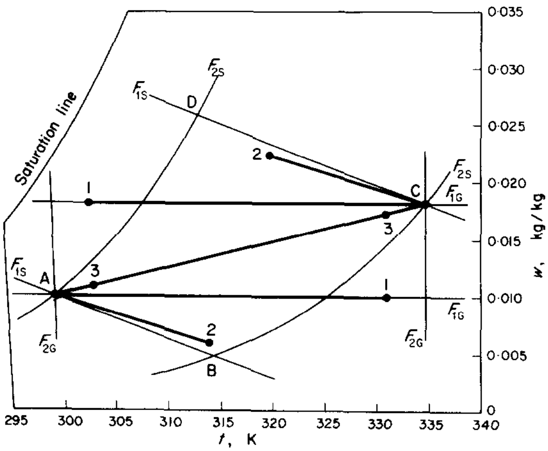

FIG. 14. Inlet and outlet states for examples plotted on F_i chart. A and C are the inlet states for periods one and two respectively. The outlet states are numbered 1 for the sensible heat regenerator, 2 for the regenerative dehumidifier and 3 for the total heat regenerator. Two sets of characteristics are shown F_{1G}, F_{2G} for glass beads and F_{1S}, F_{2S} for silica gel.

ACKNOWLEDGEMENT

The first author is grateful for the assistance of a Monash Graduate Scholarship during the course of this work.

REFERENCES

1. E. N. LIGHTFOOT, R. J. SANCHEZ-PALMA and D. O. EDWARDS, Chromatography and allied fixed-bed separation processes, *New Chemical Engineering Separation Techniques*, edited by H. M. SCHOEN, pp. 99–181. Interscience, New York (1962).
2. R. V. DUNKLE and I. L. MACLAINE-CROSS, Theory and design of rotary regenerators for air conditioning, *Mech. Chem. Engng Trans. Inst. Engrs Aust.* **6**, 1–6 (1970).
3. G. C. F. ASKER, Advantages of humidity control by adsorption dehumidifiers in spaces requiring low humidities, *Humidity and Moisture*, edited by A. WEXLER, Vol. 2, pp. 392–400. Reinhold, New York (1965).
4. G. C. F. ASKER, Heat recovery, *Air Condit. Heat. Vent.* **64** (8), 4–7 (1967).

Table 1. Data and results for examples in section 7

Quantity	Method of calculation		Sensible heat regenerator: glass beads	Regenerative dehumidifier: silica gel beads	Total heat regenerator: silica gel beads
η_{21}	data		0·900	0·900	0·900
E_{13}	data		0·002	0·050	3·0
μ	data		2540	1380	1380
$\bar{\gamma}_{11}$	Fig. 12, sections 6.2, 6.3 and 7		0	0·447	0·447
$\bar{\gamma}_{12}$	Fig. 12, sections 6.2, 6.3 and 7		0	0·447	0·447
$\bar{\gamma}_{21}$	Fig. 12, sections 6.2, 6.3 and 7		0·8	35·2	26·2
$\bar{\gamma}_{22}$	Fig. 12, sections 6.2, 6.3 and 7		0·8	25·7	26·2
E_{23}	$= E_{13}(1 + \mu\bar{\gamma}_{21})/(1 + \mu\bar{\gamma}_{11})$ from equation (17)		4·06	3·94	176
E_{12}	$= (1 + \mu\bar{\gamma}_{12})/(1 + \mu\bar{\gamma}_{11})$ equation (16) for $i = 1$		1	1	1
E_{22}	$= (1 + \mu\bar{\gamma}_{22})/(1 + \mu\bar{\gamma}_{21})$ equation (16) for $i = 2$		1	0·729	1
E_{21}	Figs. 5 and 6		9·8	4·9	9·0
E_{11}	equation (15)		9·8	4·2	9·0
η_{11}	$\eta_{11} = E_{13}$ or Fig. 5		0·002	0·050	0·888
v	equation (15) for $i = 2$	(m s^{-1})	0·745	1·132	0·777
τ	equation (17) for $i = 2$	(s)	33·6	544	13·3
η_{12}	$= E_{12}\eta_{11}$ from section 3		0·002	0·050	0·888
η_{22}	$= E_{22}\eta_{21}$ from section 3		0·900	0·656	0·900

5. W. F. RUSH and R. A. MACRISS, Munters environmental control system, *Appliance Engineer* 23–28 (1969); also *Aust. Refrig. Air Condit. Heat.*; Jubilee Issue **24**, 3A, pp. 42A/B, 50A/B, 51, 52 (1970).
6. N. A. PENNINGTON, Humidity changer for air conditioning, U.S. Patent 2,700,537 (1955).
7. R. V. DUNKLE, A method of solar air conditioning, *Mech. Chem. Engng Trans. Inst. Engrs Aust.* **1**, 73–78 (1965).
8. P. S. H. HENRY, Diffusion in absorbing media, *Proc. R. Soc.* **171A**, 215–241 (1939).
9. D. J. CLOSE and P. J. BANKS, Coupled heat and mass convection and diffusion-analogies with heat transfer, *CHEMECA* '70 (Proc. Chem. Engng Conf., Melbourne and Sydney, August, 1970) Session 6A, pp. 17–33. Butterworth of Australia, Sydney (1970).
10. P. J. BANKS, D. J. CLOSE and I. L. MACLAINE-CROSS, Coupled heat and mass transfer in fluid flow through porous media—An analogy with heat transfer. *Heat Transfer* 1970 (Proc. of the 4th Int. Transfer Conf., Versailles, Sept., 1970), Vol VII, paper CT3.1. Elsevier, Amsterdam (1970).
11. P. J. BANKS, Coupled equilibrium heat and single adsorbate transfer in fluid flow through a porous medium: Part I Characteristic potentials and specific capacity ratios, *Chem. Engng Sci.* **27**, 1143 (1972).
12. H. RHEE and N. R. AMUNDSON, An analysis of an adiabatic adsorption column: Part I Theoretical development, *Chem. Engng Jl* **1**, 241–254 (1970).
13. M. JAKOB, *Heat Transfer*, Vol. 1, p. 591. John Wiley, New York (1949).
14. C. E. BULLOCK and J. L. THRELKELD, Dehumidification of moist air by adiabatic adsorption, *Trans. Am. Soc. Heat. Refrig. Air-Condit. Engrs* **72**, Pt. 1, 301–313 (1966).
15. C. A. CHASE, D. GIDASPOW and R. E. PECK, Adiabatic adsorption in a regenerator, *Chem. Engng Prog. Symp. Ser.* **65**, 96, 34–47 (1969).
16. C. W. CHI and D. T. WASAN, Fixed bed adsorption drying, *A.I.Ch.E.Jl* **16**, 23–31 (1970).
17. H. HAUSEN, *Wärmeübertragung im Gegenstrom, Gleichstrom und Kreuzstrom.* Springer, Berlin (1950).
18. J. R. MONDT, Vehicular gas turbine periodic-flow heat exchanger solid and fluid temperature distributions, *J. Engng Pwr* **86**, 121–126 (1964).
19. J. E. JOHNSON, Regenerator heat exchangers for gas-turbines, Aeronautical Research Council R. & M. No. 2630, H.M.S.O., London (1952).
20. W. M. KAYS and A. L. LONDON, *Compact Heat Exchangers*, 2nd ed. McGraw-Hill, New York (1964).
21. T. J. LAMBERTSON, Performance factors of a periodic flow heat exchanger, *Trans. Am. Soc. Mech. Engrs* **80**, 586–592 (1958).
22. G. BIRKHOFF and S. MACLAINE, *A Survey of Modern Algebra*, 3rd ed., Chap. 9. Macmillan, New York (1965).
23. L. BRAND, *Advanced Calculus*, Chap. 4. John Wiley, New York (1955).
24. D. J. CLOSE and P. J. BANKS, Coupled equilibrium heat and single adsorbate transfer in fluid flow through a porous medium: Part II Predictions for a silica-gel air-drier using characteristic charts, *Chem. Engng Sci.* **27**, 1157 (1972).

TRANSFERTS THERMIQUES ET MASSIQUES COUPLÉS DANS DES RÉGÉNÉRATEURS. ESTIMATION UTILISANT UNE ANALOGIE AVEC UN TRANSFERT THERMIQUE

Résumé—Dans des échangeurs régénérateurs de chaleur et de masse, ou régénérateurs, la chaleur et un ou plusieurs adsorbats sont transférés cycliquement d'un courant fluide à une matrice poreuse puis de la matrice poreuse à d'autres courants fluides. Dans le modèle simple utilisé ici on décrit le transfert par un coefficient unique de transfert de film et on néglige la diffusion dans la direction de l'écoulement fluide ainsi que l'hystérésis de l'adsorption. Le comportement de ce modèle est décrit par des équations aux dérivées partielles nonlinéaires hyperboliques couplées.

Les équations sont résolues en remplaçant les variables dépendantes précédentes à savoir l'enthalpie et les adsorbats du fluide par de nouvelles variables dépendantes, les potentiels caractéristiques, et en faisant les hypothèses appropriées. Les potentiels caractéristiques et les hypothèses sont choisis de manière à ce que les équations soient divisées en un certain nombre de systèmes d'équations. Chaque système décrit le comportement d'un seul des potentiels caractéristiques et est le même que pour le transfert thermique seul excepté que le potentiel caractéristique remplace la température prise pour variable dépendante et que le rapport caractéristique de capacité spécifique remplace le rapport de chaleur spécifique matrice/fluide dans les coefficients des équations. La solution donnée dans la documentation pour des régénérateurs de chaleur peut cependant être utilisée afin de trouver une solution pour chaque potentiel caractéristique, pour l'enthalpie et les adsorbats du fluide. La solution résultante n'est qu'une approximation à cause des hypothèses faites sur le modèle.

L'application de cette méthode à des régénérateurs pour air conditionné est illustrée par un exemple numérique.

GEKOPPELTER WÄRME- UND STOFFAUSTAUSCH IN DER REGENERATOREN-BERECHNUNG MIT HILFE EINER ANALOGIE ZUM WÄRMEAUSTAUSCH

Zusammenfassung—Bein regenerativen Wärme- und Stoffaustausch in Regeneratoren werden Wärme und eine oder mehrere adsorbierte Substanzen zyklisch von einem Fluidstrom an eine poröse Spechermasse und von da zu den anderen Fluidströmen übertragen. In dem hier benutzten einfachen Modell wird der

Transportvorgang unter Verwendung eines einzigen Filmübertragungskoeffizienten beschrieben, wobei Diffusion in Strömungsrichtung und Sorptionshysteresis vernachlässigt werden. Das Modell wird durch nichtlineare, gekoppelte, hyperbolische simultane, partielle Differentialgleichungen beschrieben. Die Gleichungen werden gelöst, indem man die ursprünglichen abhängigen Variablen, die Enthalpie und die Konzentrationen der adsorbierten Substanzen im Fluid, durch neue abhängige Variable, die charakteristischen Potentiale, ersetzt und geeignete Annahmen macht. Die charakteristischen Potentiale und die Annahmen werden so gewählt, dass die Gleichungen in eine Anzahl von Gleichungssätzen zerfallen.

Jeder Satz beschreibt das Verhalten von nur einem der charakteristischen Potentiale und ist der gleiche, wie der für den reinen Wärmeaustausch, mit der Ausnahme, dass in den Koeffizienten der Gleichungen das charakteristische Potential die Temperatur als abhängige Variable ersetzt und das charakteristische Verhältnis der spezifischen Kapazitäten das Verhältnis der spezifischen Wärmen von Speichermasse und Fluid.

Die in der Literatur vorhandenen Lösungen für Wärme-Regeneratoren können deshalb dazu verwendet werden, nacheinander für jedes charakteristische Potential eine Lösung zu finden und damit auch für die Enthalpie und due Konzentrationen der adsorbierten Substanzen im Fluid. Die resultierende Lösung ist aufgrund die Annahmen eine Approximation derjenigen des Modells.

Die Anwendung dieser Methode auf Regeneratoren, wie sie in der Klimatechnik Verwendung finden, wird an einem numerischen Beispiel gezeigt.

СОВМЕСТНЫЙ ТЕПЛО- И МАССОПЕРЕНОС В РЕГЕНЕРАТОРАХ. РАСЧЕТ ПО АНАЛОГИИ С ТЕПЛООБМЕНОМ

Аннотация—В регенеративных тепло- и массообменниках, или регенераторах, перенос тепла и одного или нескольких адсорбатов происходит циклически: от жидкой струи к пористой матрице, а затем от пористой матрицы к другой струе жидкости. В простой модели, используемой в данной работе, перенос описывается с помощью одного пленочного коэффициента переноса, а диффузией в направлении течения и гистерезисом сорбции пренебрегают. Уравнения, описывающие поведение этой модели, являются системой нелинейных связанных гиперболических уравнений в частных производных.

Уравнения решаются путем замены первоначальных зависимых переменных, энтальпии и содержания адсорбата новыми зависимыми переменными и характерными потенциалами, а также с помощью соответствующих допущений. Характерные потенциалы и допущения выбираются таким образом, что уравнения делятся на несколько систем. Каждая система описывает поведение только одного характерного потенциала и совершенно эквивалентна уравнениям для чистого теплообмена, отличаясь только тем, что характерный потенциал употребляется вместо температуры в качестве зависимой переменной, а характерное отношение удельных емкостей вместо отношений теплоемкости матрицы и жидкости в коэффициентах уравнений. Поэтому решение, приведенное в литературе для тепловых регенераторов, может использоваться для нахождения решения для каждого характерного потенциала и, следовательно, для нахождения адсорбата в жидкости. Из-за сделанных допущений полученное в результате решение является приближением к решению, строго соответствующему модели.

Применение этого метода к регенераторам установок для кондиционирования воздуха иллюстрируется численным примером.